Journal of Global Optimization **17:** 259–266, 2000.

A Linear Complementarity Approach for the Non-convex Seismic Frictional Interaction between Adjacent Structures under Instabilizing Effects*

A.A. LIOLIOS
Democritus University of Thrace, Dept. Civil Engineering, Institute of Structural Mechanics and Earthquake Engineering, GR-67100 Xanthi, Greece

Abstract. The paper deals with a numerical treatment of the dynamic hemivariational inequality problem concerning the elastoplastic-fracturing unilateral contact with friction between neighboring structures under second-order geometric effects during earthquakes. The numerical procedure is based on an incremental problem formulation and on a double discretization, in space by the finite element method and in time by the Houbolt method. The generally nonconvex constitutive contact laws are piece-wise linearized, and in each time-step a nonconvex linear complementarity problem is solved with a reduced number of unknowns.

Key words: Computational contact mechanics; Dynamic hemivariational inequalities; Earthquake engineering; Nonconvex linear complementarity problem

1. Introduction

As wellknown in earthquake engineering, seismic interaction among adjacent buildings is often a main cause of damages in seismically active regions, where, due to various socioeconomic reasons, the so-called continuous building system is allowed to be applied [1, 4, 10]. Thus the numerical estimation of the interaction effects to the seismic response of such buildings is significant for their earthquake resistant design, construction and repair.

Obviously the above interaction problem is very difficult from many aspects. Mathematically this problem of pounding of buildings belongs to the inequality problems of the mathematical theory of elasticity and of the structural mechanics, where the governing conditions are equalities as well as inequalities, see e.g. Panagiotopoulos [20-21], Nitsiotas [17], Maier [12,13]. These so-called unilateral problems can be treated mathematically by the variational or hemivariational inequality concept, see e.g. Panagiotopoulos [18-21]. So, the seismic response of the interacting structures system investigated here is governed by a set of equations and inequalities, which is equivalent to a dynamic hemivariational inequality in the way used by P.D. Panagiotopoulos. As wellknown, the hemivariational inequality concept has been introduced into Mechanics and Applied Mathematics by P.D.

* This paper is dedicated to the memory of Professor P.D. Panagiotopoulos

Panagiotopoulos for first time in 1983, see [18], and constitutes now the basis of the so-called Non-Smooth Mechanics.

As regards the numerical treatment of such inequality problems in earthquake engineering and multibody dynamics, some numerical approaches have already been presented, see e.g. [1, 10, 22, 24, 30].

In the present paper, a special case of seismic building interaction is treated numerically. This case concerns the unilateral elastoplastic-softening contact between adjacent structures under second-order instabilizing effects. So, the purpose here is to estimate numerically and to control actively the influence of the interaction effects on the seismic response of the adjacent structures. The latter can be obtained by suitably adjusting the gap between the buildings (if it is possible, e.g. for new constructions), and/or the contact material behaviour (hardening or softening) according to the optimal control theory in structural analysis, see e.g. [3, 5, 8, 19, 31]. Finally, the method is applied to a civil engineering example of adjacent buildings.

2. Method of analysis

A system of only two adjacent linearly elastic structures (A) and (B) is considered here for simplicity. The extension to systems with more than two linear and/or nonlinear elastic buildings can be done in a straightforward way.

2.1. UNCOUPLED SYSTEM ANALYSIS

First the system of the two structures (A) and (B), considered as an uncoupled one, is discretized by the finite element method. So, assuming no interaction, the matrix equations of dynamic equilibrium are

$$\underline{M}_L \ddot{\underline{u}}_L + \underline{C}_L \dot{\underline{u}}_L + \underline{K}_L \underline{u}_L = -\underline{M}_L \ddot{\underline{u}}_g, \quad (L = A, B), \tag{1}$$

where $\underline{M}_L, \underline{C}_L, \underline{K}_L$ are the mass, damping and stiffness matrices, respectively; $\underline{u}(t)$ is the sought node displacement (relative to ground) vector corresponding to given ground earthquake excitation $\underline{u}_g(t)$ and appropriate initial conditions; and dots over symbols indicate time derivatives. Problem (1) can be solved by wellknown methods of Structural Dynamics.

2.2. INTERACTION SIMULATION

Let j_A and j_B be two associated nodes on the interface (joint), where unilateral frictional contact can take place during an earthquake. These nodes are considered (see Liolios [11]) as connected by two fictive unilateral constraints, normal to interface the first and tangential the second one. The corresponding force-reactions and retirement relative displacements are denoted by r_{jN}, z_{jN} and r_{jT}, z_{jT}, respectively. They satisfy in general nonconvex and nonmonotone constitutive relations of the following type (2), expressing mathematically the unilateral elastoplastic-softening contact with friction:

$$r_j(d_j) \in \partial R_j(d_j). \tag{2}$$

Here ∂ is the generalized gradient of Clarke, d the deformation and $Rj(.)$ is the superpotential function, see e.g. Panagiotopoulos [20, 21] and [7, 15, 16, 25-27]. By definition, rel. (2) is equivalent to the following hemivariational inequality:

$$R_j^{\uparrow}(d_j, e_j - d_j) \geqslant r_j(d_j).(e_j - d_j), \tag{3}$$

where $R^{\uparrow}$ denotes subderivative and e_j virtual deformation. In engineering terminology, this inequality expresses the virtual work principle holding in inequality form for unilateral constraints.

By piecewise linearizing these relations as in [9-13] we obtain the following linear complementarity conditions:

$$r_{jN} = p_{jN}(z_{jN} - g_j + w_j) + c_j z_{jN}, \tag{4}$$

$$w_j \geqslant 0, \quad r_{jN} \leqslant 0, \quad w_j.r_{jN} = 0, \tag{5a,b,c}$$

$$|r_{jT}| \leqslant f_j|r_{jN}|, \quad z_{jT}.r_{jT} = 0, \tag{6a,b}$$

$$z_{jT}.(|r_{jT}| - f_j|r_{jN}|) = 0. \tag{6c}$$

In rels. (4), (5) c_j is the damping coefficient, p_{jN} the reaction function for the normal unilateral constraint, g_j the existing normal gap and w_j a non-negative multiplier; in rels. (6) f_j is the Coulomb's friction coefficient. So, rels. (4) - (5) impose that friction phenomena (slip or adhesion) can take place only when unilateral contact occurs, i.e. when the compressive contact force r_{jN} is appeared.

2.3. COUPLED SYSTEM CONDITIONS WITH P-DELTA EFFECTS

Taking into account, now, the interaction and the second-order geometric effects (P-Delta effects), we write the incremental dynamic equilibrium conditions for the coupled system of the interacting buildings (A) and (B):

$$\underline{M}_A \Delta\underline{\ddot{u}}_A + \underline{C}_A \Delta\underline{\dot{u}}_A + (\underline{K}_A + \underline{G}_A)\Delta\underline{u}_A = -\underline{M}_A \Delta\underline{\ddot{u}}_g + \underline{T}_A \Delta\underline{r}, \tag{7a}$$

$$\underline{M}_B \Delta\underline{\ddot{u}}_B + \underline{C}_B \Delta\underline{\dot{u}}_B + (\underline{K}_B + \underline{G}_B)\Delta\underline{u}_B = -\underline{M}_B \Delta\underline{\ddot{u}}_g + \underline{T}_B \Delta\underline{r}, \tag{7b}$$

$$\underline{r} = \underline{r}_N + \underline{r}_T. \tag{7c}$$

Here $\underline{G}_A$ and $\underline{G}_B$ are the geometric stiffness matrices, by which P-Delta effects are taken into account [2, 6, 12], $\underline{T}_A$ and $\underline{T}_B$ are transformation matrices, and $\underline{r}$ is the coupling vector of the normal and tangential interaction forces, satisfying (4),(5). Appropriate initial conditions are taken into account, and so the problem consists in finding the time-dependent vectors $\{\underline{u}_A, \underline{u}_B, \underline{g}, \underline{z}, \underline{r}, \underline{w}\}$ which satisfy the rels. (2)-(7) for the given earthquake excitation $\underline{u}_g(t)$.

2.4. TIME DISCRETIZATION AND PROBLEM SOLUTION

Further the problem of rels. (2)-(7) is discretized in time. Because this problem is nonlinear – due to inequalities – the mode superposition method cannot be applied. Thus, as suggested in [28], direct time-integration methods have to be used. Here the Houbolt method is preferred to other implicit schemes and a suitable elimination of some unknowns is made. In each time-step we assume that the unilateral constraints remain either active or inactive by adjusting suitably the time-step. To compute what is happening, the procedure of Liolios [11] is applied. So, a nonconvex linear complementarity problem of the following form is eventually solved by available algorithms [14–15, 23, 29]:

$$\underline{v} \geqslant \underline{0}, \quad \underline{Dv} + \underline{d} \leqslant \underline{0}, \quad \underline{v}^T.(\underline{Dv} + \underline{d}) = 0. \tag{8}$$

Due to non-convexity of the interface behaviour (frictional unilateral contact, descending branches in the relevant stress-deformation diagrammes etc., see e.g. Fig. 1d for the numerical example of the next section), the matrix $\underline{D}$ does not correspond to a strictly positive quadratic form. But the hemivariational inequality (3), interpreted from the engineering point of view for the case of a stable system (no collapse), means that the internal virtual energy of the coupled system is greater of/or equal to external virtual work. So, using at every time-step the hemivariational inequality (3) as a stability criterion , it can be proved that for most practical applications in structural mechanics, the matrix $\underline{D}$ is a P-matrix. Thus a unique solution of the nonconvex linear complementarity problem (8) can be assured [9, 12].

2.5. INFLUENCE COEFFICIENTS

Further, we introduce the influence coefficients

$$c = \frac{Q^c - Q^u}{Q^u} \tag{9}$$

where Q is the absolutely maximum value which takes a response quantity during the seismic excitation. Index (c) is for the coupled system and index (u) for the uncoupled one (i.e. without interaction). By the influence coefficients c comparison is made between the uncoupled and the coupled cases. Thus, these coefficients show whether a structural element is overstressed or understressed due to interaction.

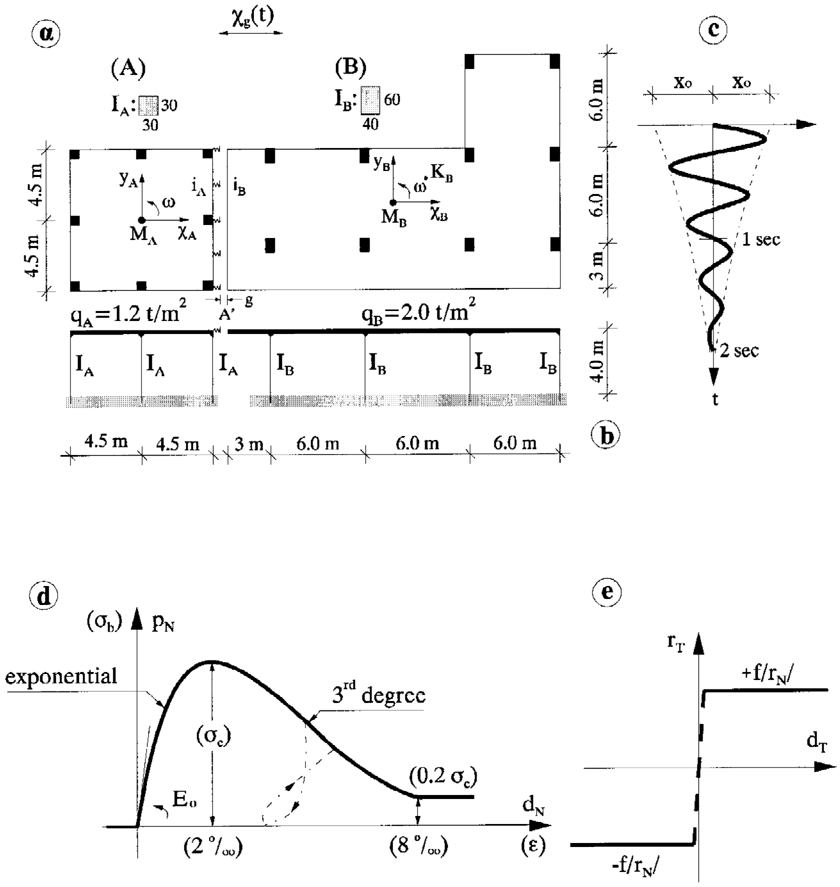

Figure 1. Numerical example: a. Plan view of the system of buildings (A) and (B); b. Vertical section view; c. Seismic ground displacement; d. Stress-deformation law for the normal unilateral constraints; e. Stress-deformation law for the tangential unilateral constraints

3. Numerical example

The system of the two one-storey buildings (A) and (B) of Fig. 1a,b are of reinforced concrete with elasticity modulus $E_b = 3{*}10^7$ KN/m^2, slab thickness 0.25 m, damping ratio 5% and beams 30/80 cm connecting the columns tops perimetrically. The columns section is 30/30 cm for (A) and 40/60 cm for (B). The stress-deformation law for the unilateral constraints normal to the interface $J - J$ is estimated by experimental results to be given as in Fig. 1d, where $\sigma_c = 18$ MPa, and the friction coefficient in Fig. 1e for the tangential constraints is estimated as $f_j = 0.40$. The system is subjected to the horizontal ground seismic excitation along the axis $x - x$, as depicted in Fig. 1c and mathematically expressed by the relation: $u_g(t) = u_0 e^{-2t} \sin(4\pi t)$, with $u_0 = 10$ mm.

Table 1. Comparison of some response quantities

Building	Quantity	Uncoupled system		Coupled system	
(A)	H_x	902	KN	784.8	KN
	H_y	0	KN	87.3	KN
	M_z	0	KNm	438.7	KNm
(B)	H_x	3071.9	KN	3302.5	KN
	H_y	588.3	KN	403.1	KN
	M_z	12161.0	KNm	11543.4	KNm

Assuming no interaction, the building (A) is symmetric from the seismic point of view and so appears transitional vibrations along the axis x-x only. On the contrary, building (B) is an asymmetric one, and appears transitional as well as torsional vibrations. So, when a seismic interaction takes place, building (A) will also appear an asymmetric response.

In Table 1, the absolutely extremum values of some response quantities, occurred during the earthquake excitation and computed by the herein presented method, are shown indicatively. These quantities, necessary for the usual aseismic design, are the horizontal forces (base shear forces) H_x and H_y and the torsional moment M_z in the mass centers of the buildings (A) and (B). The relative values are given for no interaction (uncoupled system) and for the case when frictional interaction and P-delta effects are taken into account (coupled system). As the table values show, the interaction effects in the second case are remarkable, especially as regards H_y and M_z.

4. Concluding remarks

Frictional seismic interaction under second-order effects, which is often not taken into account in the usual Civil Engineering design of adjacent buildings, can change significantly the earthquake response of such structures subjected to unilateral contact. As in the numerical example has been shown, a numerical estimation of the so caused seismic interaction effects can be obtained by the herein-presented approach. Thus, the numerical procedure is realized by using available computer codes of the finite element method and the nonlinear mathematical programming (non-convex optimization algorithms).

Certainly the most complicated task, from the earthquake engineering point of view, in all the above cases is the realistic simulation of the dynamic unilateral contact behavior. To overcome this difficulty, experimental results can be used for the rational estimation of parameters involved to simulate the interface behaviour between adjacent structures. On the other hand, the herein presented numerical approach can be used effectively to estimate numerically and to control actively the influence of the interaction on the seismic response of adjacent structures. This can be obtained by using methods of the optimal control in order to adjust the gap between the buildings and/or the contact material behaviour (hardening or softening) of the structural interface elements.

Finally, it is necessary to be emphasized that the basis of the herein presented approach is the concept of the hemivariational inequalities, introduced into Mechanics and Applied Mathematics by P.D. Panagiotopoulos, see [18–21]. These inequalities constitute now the basis for the effective treatment of many unilateral problems, and especially for the Non-Smooth Mechanics. Thus, after the sudden passing away of Professor P.D. Panagiotopoulos at 12th of August 1998, it is here proposed by the author (A. L.) that **the hemivariational inequalities** must be called from now on **the Panagiotopoulos Inequalities**.

References

1. Anagnostopoulos, S.A. and Spiliopoulos K.V. (1991), Analysis of Building Pounding due to Earthquakes. In: Krätzig W.B. et al (eds.), *Structural Dynamics*, pp. 479–484, Balkema, Rotterdam.
2. Anastasiadis, K. (1986), Méthode simplifiée de calcul du second ordre de bâtiments à étage, *Construction Metallique* 4: 43–68.
3. Baniotopoulos, C.C. (1995), Optimal control of above-ground pipelines under dynamic excitations, *Int. Jnl. Pressure Vessel & Piping* 63: 211–222.
4. Bertero, V.V. (1987), Observations on structural pounding. Proc. Intern. Conf. *The Mexico Earthquakes*, ASCE, 264–278
5. Bisbos, C. (1985), Aktive Steuerung erdbebenerregter Hochhäuser, *Z. Angew. Math. Mech. (ZAMM)* 65: T297–9.
6. Chen, W.F. and Lui, E.M. (1981), *Structural Stability*, Elsevier, New York.
7. Dem'yanov, V.F., Stavroulakis, G.E., Polyakova, L.N. and Panagiotopoulos^{+}, P.D. (1996), *Quasidifferentiability and nonsmooth modelling in mechanics, engineering and economics,* Kluwer Academic, Dordrecht, Boston, London.
8. Haslinger, J. and Neittaanmäki, P. (1996), *Finite element approximation for optimal shape, material and topology design*, Wiley, Chichester (2nd edition).
9. Klarbring, A. (1986), General contact boundary conditions and the analysis of frictional systems. *Int. J. Solids Structures*, 22: 1377–1398.
10. Liolios, A.A. (1984), *A finite-element central-difference approach to the dynamic problem of nonconvex unilateral contact between structures.* In: B. Sendov, R. Lazarov and P. Vasilevski (eds.), *Numerical Methods and Applications*, pp. 394–401. Bulgarian Acad. Sciences, Sofia.
11. Liolios, A.A. (1989), A linear complementarity approach to the nonconvex dynamic problem of unilateral contact with friction between adjacent structures. *Z. Angew. Math. Mech. (ZAMM)* 69: T 420–T 422.
12. Maier, G. (1971), Incremental Elastoplastic Analysis in the Presence of Large Displacements and Physical Instabilizing Effects, *Int. Jnl. Solids and Structures* 7: 345–372.
13. Maier, G. (1973), Mathematical programming methods in structural analysis. In: Brebbia, C. & H. Tottenham (eds.), *Variational methods in engineering, Proc. Int. Conf. Southampton University Press, Southampton,* Vol. 2, 8/1–8/32.
14. Miettinen, M., Mäkelä M.M. and Haslinger, J. (1995), On mumerical solution of hemivariational inequalities by nonsmooth optimization methods, *Journal of Global Optimization* 6: 401–425.

15. Mistakidis, E.S. and Stavroulakis, G.E. (1998), *Nonconvex Optimization in Mechanics: Algorithms, Heuristics and Engineering Applications*, Kluwer Academic, Dordrecht, Boston, London.
16. Naniewicz, Z. and Panagiotopoulos, P.D. (1995), *Mathematical Theory of Hemivariational Inequalities and Applications*, Marcel Dekker, New York.
17. Nitsiotas, G. (1971), Die Berechnung statisch unbestimmter Tragwerke mit einseitigen Bindungen, *Ingenieur-Archiv* 41: 46–60.
18. Panagiotopoulos$^{=}$, P.D., (1983), Non-convex Energy Functions. Hemivariational Inequalities and Substationarity principles, *Acta Mechanica* 48: 111–130.
19. Panagiotopoulos$^{=}$, P.D. (1984), Optimal control of structures with convex and nonconvex energy densities and variational and hemivariational inequalities, *Eng. Struct.* 6: 12–18.
20. Panagiotopoulos$^{=}$, P.D. (1985), *Inequality problems in mechanics and applications. Convex and nonconvex energy functions*, Birkhäuser Verlag, Boston-Basel-Stuttgart.
21. Panagiotopoulos$^{=}$, P.D. (1993), *Hemivariational Inequalities. Applications in Mechanics and Engineering.* Springer-Verlag, Berlin, New York.
22. Papadrakakis, M., Apostolopoulou, K., Bitzarakis, S. and Zacharopoulos, A. (1993), *A 3D model for the analysis of building pounding during earthquakes.* In: Moan, T. et al., (eds.), *Structural Dynamics-Eurodyn ’93*, pp. 85–92, Balkema, Rotterdam.
23. Pardalos, P. (1991), Global Optimization Algorithms for Linearly Constrained Indefinite Quadratic Problems, *Computers Math. Applic.* 21(6/7): 87–97.
24. Pfeiffer, F. and Glocker, Ch. (1996), *Multibody Dynamics with Unilateral Contacts*, Wiley, New York.
25. Stavroulakis, G.E. (in press), Quasi-variational inequalities. In: Floudas, C.A. and Pardalos, P.A. (Eds.), *Encyclopedia of Optimization,* Kluwer Academic, London.
26. Stavroulakis, G.E., Antes, H. and P.D. Panagiotopoulos$^{=}$ (1999), Transient elastodynamics around cracks including contact and friction, Comp. Meth. in Appl. Mech. and Enging., Special Issue: Computational Modeling of Contact and Friction, Eds.: J.A.C. Martins and A. Klarbring, 177(3/4), 427–440.
27. Stavroulakis, G.E. and Mistakidis, E.S. (in press), Nonconvex Energy Functions: Hemivariational Inequalities. In: Floudas, C.A. and Pardalos, P.A. (Eds.), *Encyclopedia of Optimization,* Kluwer Academic, London.
28. Talaslidis, D. and P.D. Panagiotopoulos$^{=}$, (1982), A linear finite element approach to the solution of the variational inequalities arising in contact problems of structural dynamics. *Int. J. Num. Meth. Enging.*, 18: 1505–1520.
29. Tzaferopoulos, M.A. and Liolios, A.A. (1997), On a Branch and Bound Algorithm for the Solution of a Family of Softening Material Problems of Mechanics with Applications to the Analysis of Metallic Structures, *Journal of Global Optimization* 11: 133–149.
30. Wolf, J.P. and Skrikerud, P.E. (1980), Mutual pounding of adjacent structures during earthquakes. *Nuclear Engin. Design* 57: 253–275.
31. Zacharenakis, E.C. (1997), On the Disturbance Attenuation and H^{∞} -Optimization in structural analysis, *Z. Angew. Math. Mech. (ZAMM)* 77: 189–195.